European qualifiers to the 2018 FIFA World Cup can be manipulated

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Ein Fehler in der ursprünglichen Versammlung der Heere ist
im ganzen Verlauf des Feldzuges kaum wieder gut zu machen.

(Helmuth Karl Bernhard von Moltke: *Taktisch-strategische Aufsätze aus den Jahren 1857 bis 1871*)

Abstract

Tournament organizers supposedly design rules such that a team cannot be better off by exerting a lower effort. It is shown that the European qualifiers to the 2018 FIFA World Cup are not strategy-proof in this sense: a team might be eliminated if it wins in the last matchday of group stage, while it advances to play-offs by playing a draw, provided that all other results do not change. An example reveals that this scenario could have happened in October 2017, after four-fifth of all matches have already been played. We present a model and identify nine incentive incompatible qualifiers to recent UEFA European Championships or FIFA World Cups. A mechanism is suggested in order to seal the way of manipulation in group-based qualification systems.

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Keywords: OR in sport; 2018 FIFA World Cup; tournament ranking; mechanism design; strategy-proofness

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1 Introduction

One important role of scientific research is to inform decision-makers about the possible properties, especially failures of different rules and formulas. It is an essential issue on the field of sport, since a bad regulation can easily lead to public outrage: one recent example was Badminton at the 2012 Summer Olympics – Women's doubles (Kendall and Lenten, 2017, Section 3.3.1). It is not an unknown phenomenon in *football*[1], too, as illustrated by Barbados vs. Grenada (1994 Caribbean Cup qualification) (Kendall and Lenten, 2017, Section 3.9.4), or the notorious 'Nichtangriffspakt (Schande) von Gijón'[2] (Kendall and Lenten, 2017, Section 3.9.1). A number of similar cases are discussed in Kendall and Lenten (2017).

These negative events may have contributed to the increasing popularity of operations research analysis of sport ranking rules (Gerchak, 1994; Wright, 2009, 2014), and to the recent application of an axiomatic approach towards sport rankings (Berker, 2014; Csató, 2017b,d; Vaziri et al., 2017). We aim to continue this research direction by analysing some qualification tournaments with respect to manipulability / strategy-proofness / incentive compatibility. If this condition doe not hold, teams[3] might have a possibility to gain by performing worse in certain matches.

Specifically, the qualifiers to two prominent football competitions, to the UEFA European Championships and FIFA World Cups in the European Zone will be discussed from this point of view. We get a negative result as the monotonicity of rankings for each separate subtournaments of the qualifier is seemingly not enough to guarantee strategy-proofness for the whole qualification system.

The root of the problem has still been revealed by a column in the case of the European qualification for the 2014 FIFA World Cup in Brazil (Dagaev and Sonin, 2013), and has been described by Dagaev and Sonin (2017) in a sentence: '*Two months before the end of the tournament, with 80% of games completed, there still was a scenario under which a team might need to achieve a draw instead of winning to go to Brazil.*'[4]

This paper outlines a similar scenario for 2018 FIFA World Cup qualification (UEFA) by showing that a team might be eliminated if it wins in the last matchday of group stage, but it advances to play-offs by playing a draw, provided that all other results do not change. Crucially, the example takes the outcome of matches played before October 2017 and the appearance of the first version of the current paper (Csató, 2017a) as given. After a detailed presentation of the particular example, we formalize the problem of group-based qualification tournaments. A pair of theorems lists the conditions of strategy-proofness and incentive incompatibility, respectively. They are applied to identify nine recent qualification systems, which open a way to manipulation. Finally, we suggest a mechanism for tournament organizers in order to address the problem.

The rest of the article is organized as follows. In Section 2, we discuss related literature. Section 3 describes the real-world example, the European section of the 2018 FIFA World Cup qualification. The incentive incompatibility of this qualification is proved in Section 4. The main model is presented and analysed in Section 5, and is applied to examine the

[1] Throughout the paper, we take the European meaning of football, rather than the US meaning.

[2] Kendall and Lenten (2017) use the term 'Shame of Gijón', and Wikipedia calls it 'Disgrace of Gijón'.

[3] The word team is used because of the example, but they can also be players in other settings.

[4] We have written the first version of the paper (Csató, 2017a) without knowing about Dagaev and Sonin (2013) or Dagaev and Sonin (2017). While it is not an excuse for the originality of the current research, this fact indicates that the failure of qualification rules is almost obvious. It is worth to note that Dagaev and Sonin (2017) also build on a 'borrowed' real-world example.

strategy-proofness of several qualification systems in Section 6. Section 7 formulates policy implications for tournament organizers, and Section 8 concludes.

The paper is written both for the public and the scientific community. Decision-makers not interested in or not familiar with the theoretical background of manipulation can skip Sections 2, 5 and maybe 2 as well as 3, in order to focus on the example presented in Section 4 and the possible remedies in Section 7.

2 Related work

Ranking in sport tournaments is closely related to the problem of preference aggregation. In social choice theory, a number of articles have dealt with the axiomatic properties of different aggregation rules (see, e.g., Arrow (1950); Rubinstein (1980); Chebotarev (1994); Chebotarev and Shamis (1998); van den Brink and Gilles (2003); Slutzki and Volij (2005, 2006); Altman and Tennenholtz (2008); van den Brink and Gilles (2009); González-Díaz et al. (2014); Csató (2017c)). For example, the well-known Gibbard-Satterthwaite (Gibbard, 1973; Satterthwaite, 1975) and the more general Duggan-Schwartz (Duggan and Schwartz, 2000) theorems state that fairness is not compatible with strategy-proofness, that is, if a voting rule is fair, there always exists a voter who can achieve a better outcome by tactical voting.

Some recent works have analysed incentive (in)compatibility of sport ranking rules. Stanton and Williams (2013) investigate double-elimination tournaments (a competition where no participant is eliminated until it lost two matches) and show that they are vulnerable to manipulation by a coalition of players who can improve their chance of winning by throwing matches. Russell and Walsh (2009) and Schneider et al. (2016) also discuss manipulation by coalitions through a collusion between several teams. Pauly (2014) develops a mathematical model of strategic manipulation in round-robin subtournaments and derives an impossibility theorem. Vong (2017) considers the strategic manipulation problem in multistage tournaments and shows that it is necessary to allow only the top-ranked player to qualify from each group in order to guarantee that all of them exert full effort. Dagaev and Sonin (2017) prove that tournament systems, consisting of multiple round-robin and knock-out tournaments with noncumulative prizes, are characteristically incentive incompatible. Brams and Ismail (2016) and Brams et al. (2016) address the strategy-proofness of certain sporting rules, too. Russell (2010) studies the complexity of manipulation strategies in knock-out and round-robin tournaments and presents some algorithms which are able to identify with high accuracy whether a coalition is manipulating the tournament. Lasek et al. (2016) suggest some strategies for improving a team's position in the official ranking of international football teams compiled by FIFA.

Some occurrences of incentive incompatibility (like the Badminton scandal of the 2012 Olympics, which probably inspired Pauly (2014) and Vong (2017)) are based on the fact that being ranked lower in the qualification tournament might lead to a more preferred competitor in the following knock-out stage. In this paper, similarly to Dagaev and Sonin (2017), we do not address this probabilistic aspect of manipulation (note that facing another competitor only means advantage in expected terms), and discuss only cases when a team is *strictly better off* by exerting a lower effort.

3 2018 FIFA World Cup qualification (UEFA)

2018 FIFA World Cup qualification (UEFA) is a short denomination of the European section of the 2018 FIFA[5] World Cup qualification, the qualifier of national association football teams which are members of UEFA[6] to the 2018 FIFA World Cup, to be held in Russia.[7] With the admission of Gibraltar and Kosovo as FIFA members in May 2016, 54 teams compete in this qualification for 13 slots in the final tournament (Russia automatically qualifies as a host).

The qualifying format was confirmed by the UEFA Executive Committee meeting on 22-23 March 2015 in Vienna. The qualification structure is as follows:

- Group stage (first round): Nine groups of six teams each, playing home-and-away round-robin matches. The winners of each group qualify for the 2018 FIFA World Cup, and the eight best runners-up advance to play-offs (second round).
- Play-offs (second round): The eight best second-placed teams from the group stage play home-and-away matches over two legs. The four winners qualify for the 2018 FIFA World Cup.

We focus on the first round. FIFA (2016, Article 20.4a) specifies this stage.

The matches shall be played in accordance with one of the following three formats:

a) in groups composed of several teams on a home-and-away basis, with three points for a win, one point for a draw and no points for a defeat (league format);

Tie-breaking rules in the groups are detailed in FIFA (2016, Article 20.6).

In the league format, the ranking in each group is determined as follows:

a) greatest number of points obtained in all group matches;

b) goal difference in all group matches;

c) greatest number of goals scored in all group matches.

If two or more teams are equal on the basis of the above three criteria, their rankings shall be determined as follows:

d) greatest number of points obtained in the group matches between the teams concerned;

e) goal difference resulting from the group matches between the teams concerned;

f) greater number of goals scored in all group matches between the teams concerned;

[5] FIFA stands for *Fédération Internationale de Football Association*, French for International Federation of Association Football, which is the international governing body of association football, futsal, and beach soccer.

[6] UEFA stands for *Union of European Football Associations*, the administrative body for association football in Europe. However, several UEFA member states are primarily or entirely located in Asia. It is one of the six continental confederations of world football's governing body FIFA.

[7] This section is mainly based on the Wikipedia page of 2018 FIFA World Cup qualification (UEFA). We will cite only those official documents which concern the ranking of teams.

g) the goals scored away from home count double between the teams concerned (if the tie is only between two teams).

h) fair play points system in which the number of yellow and red cards in all group matches is considered according to the following deductions:

– *first yellow card:*	*minus 1 point*
– *second yellow card/indirect red card:*	*minus 3 points*
– *direct red card:*	*minus 4 points*
– *yellow card and direct red card:*	*minus 5 points;*

i) drawing of lots by the FIFA Organising Committee.

Strangely, FIFA (2016, Article 20.6) does not state explicitly that greater goal differences and fair play points are preferred.[8] Choice of the eight best second-placed teams is not addressed in this document. FIFA (2016, Article 20.8) only describes that

Should the best second- or third-placed team within a group stage qualify for the next stage or for the final competition, the criteria to decide such best second- or third-placed team shall depend on the competition format and shall require the approval of FIFA following proposals from the confederations.

Unfortunately, we were not able to find the relevant regulation of UEFA. But, according to a FIFA (FIFA, 2017) Media Release (reinforced by an UEFA news (UEFA, 2016)),

the eight best runners-up will be decided by ranking criteria as stated in the 2018 FIFA World Cup Regulations, namely points, goal difference, goals scored, goals scored away from home and disciplinary ranking, with the results against teams ranked 6th not being taken into account.

Since head-to-head results are nonexistent in the comparison of runners-up, the ranking of second-placed teams strictly follow tie-breaking in groups, with the crucial difference of discarding two matches played against the last team of the group.[9]

[8] The purpose of mixing words *greater* and *greatest* is not clear for us. Note also the inconsistent use of the word *all* before group matches.

[9] However, it seems that there were some controversy around the ranking of second-placed teams. According to our knowledge, FIFA and UEFA did not publish the ranking of second-placed teams *before* the end of group stage. The Spanish Wikipedia page of 2018 FIFA World Cup qualification (UEFA) ranked the runners-up on the basis of all matches played even on 12 September 2017. Most Wikipedia pages of the qualification, like the English, French, or Hungarian have placed Sweden on the 6th and Montenegro on the 7th position on 12 September 2017, after eight matchdays were played. On the other hand, Montenegro was the 6th and Sweden the 7th in German and Italian. As one can check in Tables 2 and A.4, Sweden and Montenegro had the same goal difference (+3) and number of goals scored (10) at that time. Furthermore, both teams scored 4 goals away from home. There is a difference in the goals against them at home, other teams of the group (without the last) had scored 2 goals in Sweden and 3 in Montenegro. It may be a weak argument to rank Sweden higher, nevertheless, in the lack of exact rules, we are not sure. It is also possible that disciplinary points count, but then with or without the matches against the last team? Anyway, it is rather an academic issue as it obviously does not influence which teams advance to play-offs.

Another confederation of FIFA, the AFC (Asian Football Confederation) has published a Media Release (AFC, 2015) on the ranking of runners-up.[10] AFC (2015, Case 2) also provides an illustration on how to calculate a ranking of second-placed teams when some group matches are discarded.

We will see that this, seemingly minor, modification in the comparison of second-placed teams has some unintended consequences regarding manipulation.

4 How 2018 FIFA World Cup qualification (UEFA) can be manipulated?

In this section we will present a possible manipulation of the European qualifiers to the 2018 FIFA World Cup. Matches of the first eight matchdays – to be played between 4 September 2016 and 5 September 2017 – are assumed to be given as they were known when the first version of the paper (Csató, 2017a) was made publicly available.[11]

Theorem 4.1. *It might still happen after four-fifth of all matches are over that 2018 FIFA World Cup qualification (UEFA) can be manipulated by Bulgaria playing a draw instead of a win against Luxembourg in the last matchday, on 10 October 2017.*

Proof. We provide an example by generating results for the last two matchdays, to be played between 5 October 2017 and 10 October 2017.[12] Eight groups are shown in the Appendix: Table A.1 presents Group B; Table A.2 presents Group C; Table A.3 presents Group D; Table A.4 presents Group E; Table A.5 presents Group F; Table A.6 presents Group G; Table A.7 presents Group H; and Table A.8 presents Group I.

Since the manipulation may occur Group A, it is discussed in detail here. Table 1 shows a possible scenario in this group. Note that some hypothetical results of Table 1.b may be unreasonable, like Belarus defeating Netherlands by 7-0. They are necessary to create the appropriate conditions for manipulation. Nevertheless, this set of match results has at least positive probability to be realized after eight matchdays are over.

On the basis of standings in Group A-I, runners-up are ranked in Table 2. Only the eight best second-places team advance to play-offs, hence Bulgaria is eliminated.

However, consider what happens if Bulgaria plays a draw of 1-1 against Luxembourg in the last matchday, which takes place on 10 October 2017 in Group A. It is clear that this change worsen Bulgaria's standing in the group. Nevertheless, it still remains on the second place with 16 points as both Bulgaria and Sweden would have the same goal difference (+4) with Bulgaria scoring more goals in all group matches (22 vs. 18) in the

[10] Second-placed teams should be ranked in the second round of the Asian section of the 2018 FIFA World Cup qualification, organized for national teams which are members of AFC. AFC (2015) lists the following criteria as tie-breaking rules for the comparison of runners-up: *greatest number of points obtained from group matches; goal difference in group matches; greatest number of goals scored in group matches; fewer number of points calculated according to the number of yellow and red cards received by the team; drawing of lots.* Number of goals scored away from home does not appear among the criteria and the preferred direction of goal difference is not specified, although it is provided for fair play points in contrast to FIFA (2017, Article 20.6).

[11] Perhaps the best summary of 2018 FIFA World Cup qualification (UEFA) is its Wikipedia page, too. However, a national team in Group G was referred to as Macedonia (at least on 12 September 2017), while its official name used by FIFA and UEFA is FYR Macedonia, as the country was admitted by United Nations under the provisional description *the former Yugoslav Republic of Macedonia.*

[12] It is worth to note that all teams play one match home and one away in the last two matchdays, which is not necessarily true for two subsequent matchdays.

Table 1: 2018 FIFA World Cup qualification – UEFA Group A

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	France	—	2-1	4-0	4-1	0-0	10 Oct
2	Sweden	2-1	—	1-1	3-0	7 Oct	4-0
3	Netherlands	0-1	10 Oct	—	3-1	5-0	4-1
4	Bulgaria	7 Oct	3-2	2-0	—	4-3	1-0
5	Luxembourg	1-3	0-1	1-3	10 Oct	—	1-0
6	Belarus	0-0	0-4	7 Oct	2-1	1-1	—

(b) Hypothetical match results of the last two matchdays

Last row shows an alternative result, obtained if Bulgaria manipulates

Date	Home team	Away team	Result
7 October 2017	Sweden	Luxemburg	0-4
7 October 2017	Belarus	Netherlands	7-0
7 October 2017	Bulgaria	France	8-0
10 October 2017	France	Belarus	1-0
10 October 2017	Luxemburg	Bulgaria	0-1
10 October 2017	Netherlands	Sweden	3-0
10 October 2017*	Luxemburg*	Bulgaria*	1-1*

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drown; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.

Last but one row contains the second-placed team's benchmark results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Last row contains the second-placed team's alternative results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017), obtained if Bulgaria manipulates.

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	France	6	2	2	16	13	3	**20**
2	Bulgaria	6	0	4	22	17	5	**18**
3	Sweden	5	1	4	18	14	4	**16**
4	Netherlands	5	1	4	19	18	1	**16**
5	Belarus	2	2	6	11	17	-6	**8**
6	Luxembourg	2	2	6	11	18	-7	**8**
2	Bulgaria	4	0	4	17	14	3	**12**
2*	Bulgaria*	4*	1*	3*	20*	16*	4*	**13***

alternative scenario. On the other hand, Luxembourg overtakes Belarus thanks to its newly obtained draw since it has the same goal difference (-6) with more goals scored (12 vs. 11). In the ranking of second-placed teams, matches against the last team are

Table 2: 2018 FIFA World Cup qualification – Ranking of second-placed teams
Pos = Position; W = Won; D = Drown; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points.
Since matches played against the 6th team in each group are discarded (FIFA, 2017), all teams have played 8 matches taken into account.
Last row contains Bulgaria's alternative results, obtained if it manipulates.

Pos	Team	Group	W	D	L	GF	GA	GD	**Pts**
1	Portugal	B	6	1	1	23	5	18	**19**
2	Italy	G	6	1	1	14	8	6	**19**
3	Northern Ireland	C	4	2	2	9	3	6	**14**
4	Wales	D	3	5	0	8	5	3	**14**
5	Turkey	I	4	2	2	8	8	0	**14**
6	Slovakia	F	4	1	3	11	5	6	**13**
7	Greece	H	3	4	1	8	4	4	**13**
8	Montenegro	E	3	3	2	12	6	6	**12**
9	Bulgaria	A	4	0	4	17	14	3	**12**
7*	Bulgaria*	A	4*	1*	3*	20*	16*	4*	**13***

discarded. Consequently, Bulgaria would have 13 points, placing it seventh among the runners-up according to Table 2 (it has the same goal difference as Greece with more goals scored). Thus Bulgaria would advance to play-offs instead of Montenegro if it would allow a goal for Luxembourg. □

According to the example presented, there exists a set of match results (even after eight matchdays are over) such that Bulgaria advances to play-offs instead of being eliminated if it concedes a goal in its last match, provided that all other results are fixed. Montenegro is eliminated only as a consequence this unfair act, so it would have a strong argument to protest against the current rules applied by FIFA and UEFA.

It is worth to note that the example used for the proof of Theorem 4.1 is robust with respect to Groups B-I. If one considers the actual match results for these groups (still known at this moment) instead of our hypothetical ones, Slovakia is the worst second-placed teams with 12 points and a goal difference of +5 among the runner-ups. Bulgaria is still eliminated by winning against Luxembourg according to Table 1.c, but is advanced to play-offs if it plays a draw of 1-1. Manipulation mainly depends on the events in Group A.

5 Theoretical background

In the following, we build a model for the group stage of a qualification, where groups are home-and-away round-robin tournaments.

Definition 5.1. *Home-and-away round-robin tournament*: Let X be a nonempty finite set of at least two teams, $x, y \in X$ be two teams and $v : X \times X \to \{(v_1; v_2) : v_1, v_2 \in \mathbb{N}\} \cup \{—\}$ be a function such that $v(x, y) = —$ if and only if $x = y$. The pair (X, v) is called a *home-and-away round-robin tournament*.

In a home-and-away round-robin tournament, any two teams play each other once at home and once at away. Function v describes game results with the number of goals scored by home and away teams, respectively.

Definition 5.2. *Ranking in home-and-away round-robin tournaments*: Let $\mathcal{X}$ be the set of home-and-away round-robin tournaments with a set of teams X. A *ranking method* S is a function that maps any characteristic function v of $\mathcal{X}$ into a strict order $S(v)$ on the set X.

Let (X, v) be a home-and-away round-robin tournament, $S(v)$ be its ranking and $x, y \in X$, $x \neq y$ be two different teams. x is ranked higher (lower) than y if and only if $x \succ_{S(v)} y$ ($x \prec_{S(v)} y$).

Let $x, y \in X$, $x \neq y$ be two different teams and $v(x, y) = (v_1(x, y); v_2(x, y))$. It is said that team x wins over team y if $v_1(x, y) > v_2(x, y)$ (home) or $v_1(x, y) < v_2(x, y)$ (away), team x loses to team y if $v_1(x, y) < v_2(x, y)$ (home) or $v_1(x, y) > v_2(x, y)$ (away) and teams x draws with team y if $v_1(x, y) = v_2(x, y)$.

Definition 5.3. *Number of points*: Let (X, v) be a home-and-away round-robin tournament and $x \in X$ be a team. Denote by $N_v^w(x)$ the number of wins and by $N_v^d(x)$ the number of draws of team x in (X, v), respectively. The *number of points* of team x is $s_v(x) = \alpha N_v^w(x) + N_v^d(x)$ such that $\alpha \geq 2$.

In other words, a win means α points, a draw means 1 point and a loss means 0 points.

Number of points does not necessarily give a strict order on the set of teams, so some tie-breaking rules should be introduced.

Definition 5.4. *Goal difference*: Let (X, v) be a *home-and-away round-robin tournament* and $x \in X$ be a team. The *goal difference* of team x is

$$gd_v(x) = \sum_{y \in X, y \neq x} (v_1(x, y) - v_2(x, y)) + \sum_{y \in X, y \neq x} (v_2(y, x) - v_1(y, x)).$$

Goal difference is the difference of the number of goals scored for team x and the number of goals scored against team x.

Definition 5.5. *Head-to-head results*: Let (X, v) be a home-and-away round-robin tournament and $x \in X$ be a team. Denote by $L \subseteq X \setminus \{x\}$ a set of teams.

The *head-to-head number of points* of team x with respect to L in (X, v) is

$$\begin{aligned} s_v^L(x) \quad &= \quad \alpha\left(\left|\{y \in L : v_1(x, y) > v_2(x, y)\}\right| + \left|\{y \in L : v_1(y, x) < v_2(y, x)\}\right|\right) + \\ &\quad + \left|\{y \in L : v_1(x, y) = v_2(x, y)\}\right| + \left|\{y \in L : v_1(y, x) = v_2(y, x)\}\right| \end{aligned}$$

The *head-to-head goal difference* of team x with respect to L in (X, v) is

$$gd_v^L(x) = \sum_{y \in L} (v_1(x, y) - v_2(x, y)) + \sum_{y \in L} (v_2(y, x) - v_1(y, x)).$$

Definition 5.6. *Monotonicity of group ranking*: Let $\mathcal{X}$ be the set of home-and-away round-robin tournaments with a set of teams X, and S be a ranking method. S is *monotonic* if for any characteristic function v and for any different teams $x, y \in X$, $x \neq y$:

1. $s_v(x) > s_v(y) \Rightarrow x \succ_{S(v)} y$;
2. $s_v(x) = s_v(y)$, $gd_v(x) > gd_v(y)$ and $s_v^L(x) > s_v^L(z)$ or $s_v^L(x) = s_v^L(z)$ and $gd_v^L(x) > gd_v^L(y)$ for all $z \in L$ where L is the set of all teams not ranked lower or higher than x by criterion 1 and the recursive application of criterion 2 $\Rightarrow x \succ_{S(v)} y$.

Monotonicity implies that (a) a team should be ranked higher if it has a greater number of points (criterion 1); (b) a team should be ranked higher compared to another with the same number of points, an inferior goal difference and worse head-to-head results against all teams currently ranked equally (criterion 2). Monotonicity still does not imply that the ranking is unique. The complexity of the definition is necessary in order to cover the different tie-breaking rules recently applied by FIFA (goal difference) and UEFA (recursive head-to-head results). Berker (2014, Table 6) gives a short overview of them.

Definition 5.7. *Group-based qualifier*: A *group-based qualifier* $\mathcal{T}$ consists of k groups of home-and-away round-robin tournaments with the set of teams X^1, X^2, ..., X^k.

Definition 5.8. *Allocation rule of a group-based qualifier*: An *allocation rule* of a group-based qualifier $\mathcal{T}$ is a function $\mathcal{R} : \mathcal{X}^1 \times \mathcal{X}^2 \times \cdots \times \mathcal{X}^k \times \cup_{i=1}^n X^i \to \{0; 1; 2\}$.

Consider a group-based qualifier $\mathcal{T}$, its allocation rule $\mathcal{R}$, a set of group results $V = \left\{v^1, v^2, \dots, v^k\right\}$ and a team $x \in \cup_{i=1}^k X^i$. Team x is said to be (a) directly qualified if $\mathcal{R}(V, x) = 2$; (b) advanced to the next round with a chance to qualify if $\mathcal{R}(V, x) = 1$; (c) eliminated if $\mathcal{R}(V, x) = 0$.

Definition 5.9. *Qualification system*: The pair $(\mathcal{T}, \mathcal{R})$ of a group-based qualifier $\mathcal{T}$ and its allocation rule $\mathcal{R}$ is a *qualification system*.

In order to fit 2018 FIFA World Cup qualification (UEFA) into this model, the allocation rule is allowed to compare teams from different groups in an *extra group*.

Definition 5.10. *Extra group function*: Let $(\mathcal{T}, \mathcal{R})$ be a qualification system. *Extra group function* $\mathcal{G}$ associates to any set of group results $V = \left\{v^1, v^2, \dots, v^k\right\}$ a set of teams $X^{k+1} \subseteq \cup_{i=1}^k X^i$ and a set $X_x^i \subseteq X^i \setminus \{x\}$ for each $x \in X^{k+1}$.

Definition 5.11. *Extra group ranking*: Let $(\mathcal{T}, \mathcal{R})$ be a qualification system and $\mathcal{G}$ be an extra group function. An *extra group ranking method* Q is a function that maps any set of group results $V = \left\{v^1, v^2, \dots, v^k\right\}$ into a strict order on the set X^{k+1}.

Definition 5.12. *Ranking in extra group*: Let $(\mathcal{T}, \mathcal{R})$ be a qualification system and $\mathcal{G}$ be an extra group function. Then the *number of points in the extra group* of team $x \in X_{k+1}$ is

$$\begin{aligned} s_{\mathcal{G},V}^{k+1}(x) &= \alpha \left(\left| \left\{ y \in X_x^i : v_1^i(x,y) > v_2^i(x,y) \right\} \right| + \left| \left\{ y \in X_x^i : v_1^i(y,x) < v_2^i(y,x) \right\} \right| \right) + \\ &\quad + \left| \left\{ y \in X_x^i : v_1^i(x,y) = v_2^i(x,y) \right\} \right| + \left| \left\{ y \in X_x^i : v_1^i(y,x) = v_2^i(y,x) \right\} \right| \end{aligned}$$

The *goal difference in the extra group* of team $x \in X_{k+1}$ is

$$gd_{\mathcal{G},V}^{k+1}(x) = \sum_{y \in X_x^i} \left(v_1^i(x,y) - v_2^i(x,y) \right) + \sum_{y \in X_x^i} \left(v_2^i(y,x) - v_1^i(y,x) \right).$$

Note that teams of X^{k+1} have not necessarily played against each other (head-to-head results may be unknown) since there are no further matches in the extra group, but their number of points and goal difference can be defined on the basis of certain group matches.

Definition 5.13. *Monotonicity of extra group ranking*: Let $(\mathcal{T}, \mathcal{R})$ be a group-based qualification system and $\mathcal{G}$ be an extra group function. Extra group ranking Q is said to be *monotonic* if for any set of group results $V = \{v_1, v_2, \dots, v_k\}$ and for any different teams $x, y \in X^{k+1}$, $x \neq y$:

1. $s^{k+1}_{\mathcal{G},V}(x) > s^{k+1}_{\mathcal{G},V}(y) \Rightarrow x \succ_{Q(V)} y$;

2. $s^{k+1}_{\mathcal{G},V}(x) = s^{k+1}_{\mathcal{G},V}(y)$ and $gd^{k+1}_{\mathcal{G},V}(x) > gd^{k+1}_{\mathcal{G},V}(y) \Rightarrow x \succ_{Q(V)} y$.

Definition 5.13 is more simple than Definition 5.6 due to the lack of head-to-head results in the extra group.

Definition 5.14. *Fairness of an allocation rule*: Let $(\mathcal{T}, \mathcal{R})$ be a group-based qualification system. Allocation rule $\mathcal{R}$ is *fair* if:

- there exists a common monotonic ranking S in each group such that $x, y \in X^i$, $1 \leq i \leq k$ and $x \succ_{S(v_i)} y$ implies $\mathcal{R}(V, x) \geq \mathcal{R}(V, y)$;
- there exists an extra group function $\mathcal{G}$ such that $x, y \in X^{k+1}$ implies $|X^i_x| = |X^i_y|$;
- there exists a monotonic extra group ranking Q such that $x, y \in X^{k+1}$ and $x \succ_{Q(V)} y$ implies $\mathcal{R}(V, x) \geq \mathcal{R}(V, y)$.

The idea behind a fair allocation rule is straightforward. Application of a monotonic ranking in groups ensures that teams have no incentive to exert a lower effort during a match since they cannot achieve a higher position in the group by deliberately playing worse. This should also be true in the extra group, hence extra group ranking Q is required to satisfy monotonicity. The second condition is responsible for fairness in the comparison of teams from different groups: their number of matches considered in the extra group should be the same, otherwise number of scores is not a good measure of performance as it cannot decrease if a team plays more matches.

Perhaps these ideas have inspired the decision-makers of FIFA and UEFA.

Definition 5.15. *Manipulation*: Consider a qualification system $(\mathcal{T}, \mathcal{R})$ and a set of group results $V = \left\{ v^1, v^2, \dots, v^k \right\}$. A team $x \in X^i$ can *manipulate* the qualification system $(\mathcal{T}, \mathcal{R})$ if there exists a set of group results $\bar{V} = \left\{ v^1, v^2, \dots, \bar{v}^i, \dots, v^k \right\}$ such that $\bar{v}^i_2(x, y) \geq v^i_2(x, y)$ and $\bar{v}^i_1(y, x) \geq v^i_1(y, x)$ for all $y \in X^i$ and $\mathcal{R}(V, x) < \mathcal{R}(\bar{V}, x)$.

Manipulation means that team x can improve its position with respect to qualification by letting its opponents to score more goals.

Definition 5.16. *Strategy-proofness*: A qualification system $(\mathcal{T}, \mathcal{R})$ is called *strategy-proof* if there exists no set of group results $V = \left\{ v^1, v^2, \dots, v^k \right\}$ such that a team can manipulate it.

Our main results concern the strategy-proofness of qualification systems with a fair allocation rule.

Theorem 5.1. *Let $(\mathcal{T}, \mathcal{R})$ be a qualification system such that $\mathcal{R}$ is a fair allocation rule and the following conditions hold:*

- *the number of groups is at least $k \geq 2$;*
- *there is a difference in the allocation of teams in the extra group, that is, if $x \in X^{k+1}$, then there exists $z \in X^{k+1}$ such that $\mathcal{R}(V, x) \neq \mathcal{R}(V, z)$;*
- *there exists a team $x \in X^i \cap X^{k+1}$ such that $\left| \left\{ y \in X^i : x \succ_{S(v^i)} y \right\} \right| \geq \ell + 1$ and $z \in X^i \setminus \{x\}$ if and only if $\ell = \left| \left\{ y \in X^i : z \succ_{S(v^i)} y \right\} \right| \geq 1$.*

Then qualification system $(\mathcal{T}, \mathcal{R})$ does not satisfy strategy-proofness.

According to the second requirement of Theorem 5.1, teams of the extra group are distinguished by the allocation rule. The third condition means that if a team is considered in the extra group such that its matches played against the lowest ranked ℓ teams in its group are discarded, then at least $\ell + 1$ teams ranked lower than it can be found in the group.

Proof. An example is presented where a team can manipulate a qualification system that satisfies all criteria of Theorem 5.1.

Table 3: Group 1 of Example 5.1

GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points.
Last but one row contains the group winner's benchmark results, adjusted for ranking in the extra group (matches played against the last team are discarded) according to the allocation rule $\mathcal{R}$.
Last row contains the group winner's alternative results, adjusted for ranking in the extra group (matches played against the last team are discarded) according to the allocation rule $\mathcal{R}$, obtained if team a manipulates.

Position	Team	a	b	c	GF	GA	GD	**Pts**
1	a	—	3-0	4-0	7	2	5	$2\alpha + 1$
2	b	2-0	—	1-0	3	6	-3	2α
3	c	0-0	3-0	—	3	4	-1	$\alpha + 1$
1	a	—	—	—	3	2	1	α
1*	a*	—	—	—	4*	1*	3*	α*

Example 5.1. Let $k = 2$, $X^1 = \{a, b, c\}$ and $X^2 = \{d, e\}$.

Consider the fair allocation rule $\mathcal{R}$ such that $x \in X^3$ if and only if $x \succ_{S(v)} y$ for all $x, y \in X^i$, $x \neq y$, $X^i_x = \left\{ y \in X^i \setminus \{x\} : \left| \left\{ z \in X^i : z \succ_{S(v)} y \right\} \right| = 1 \right\}$ and

$$\mathcal{R}(V, x) = \begin{cases} 0 & \text{if there exists a team } y \in X^i : x \prec_{S(v)} y \\ 0 & \text{if there exists a team } y \in X^{k+1} : x \prec y \\ 2 & \text{otherwise} \end{cases}$$

$\mathcal{R}$ says that teams not winning their group are eliminated, the extra group consists of the two remaining teams and the one ranked higher by Q – after the matches of the first team against the third in Group 1 are discarded – qualifies.

A possible set of results in Group 1 is shown in Table 3. Team a should be the first since it has the most points (see criterion 1 of a monotonic group ranking method), and it is considered in the extra group with α points and a goal difference of $+1$ after discarding the two matches against team c, the last in Group 1 due to criterion 1 of a monotonic group ranking method (see the last but one row of Table 3).

There are only two matches to be played in Group 2. Let v^2 be given such that $v^2(d, e) = (3; 0)$ and $v^2(e, d) = (1; 0)$. Then team d should be the first (see criterion 2 of a monotonic group ranking method) and would be considered in the extra group with α points and a goal difference of $+2$. Consequently, $\mathcal{R}(V, a) = 0$ and $\mathcal{R}(V, d) = 1$ due to criterion 2 of a monotonic extra group ranking method.

However, examine what happens when $\bar{v}^1(c, a) = (1; 0)$. Then teams a, b and c have 2α points, but team a has the same head-to-head number of points and better head-to-head goal difference compared to either b or c, thus it is ranked first. Furthermore, team c has the same head-to-head number of points and better head-to-head goal difference compared to b, so it would be the second. This ranking is based on criterion 2 of a monotonic group ranking method. Consequently, team a is considered with α points and a goal difference of $+3$ in the extra group (see the last row of Table 3), thus $\mathcal{R}(\bar{V}, a) = 1$ and $\mathcal{R}(\bar{V}, d) = 0$ due to criterion 2 of a monotonic extra group ranking method.

To summarize, team a has an opportunity to manipulate this qualification structure under a set of group results V, so it is not strategy-proof.

Example 5.1 has the least possible number of teams, three in the first and two in the second group. It is clear that the number of groups and the number of teams in them as well as parameter ℓ can be increased without changing the essence of the counterexample. □

Remark 5.1. 2018 FIFA World Cup qualification (UEFA), discussed in Section 3, fits into the model presented above. The number of groups is $k = 9$ and the allocation rule $\mathcal{R}$ is as follows:

- S is monotonic because number of points is the first and goal difference is the second tie-breaker in groups;
- Q is monotonic because number of points is the first and goal difference is the second tie-breaker in the extra group;
- the first-placed team in each group qualifies: $\mathcal{R}(V, x) = 2$ for all $x \in X^i$ if and only if $\nexists y \in X^i : y \succ_{S(v)} x$;
- the third-, fourth-, fifth- and sixth-placed teams in each group are eliminated: $\mathcal{R}(V, x) = 0$ for all $x \in X^i$ if $\left|\left\{y \in X^i : y \succ_{S(v^i)} x\right\}\right| \geq 2$;
- the extra group consists of the second-placed teams: $x \in X^{k+1}$ if and only if $\left|\left\{y \in X^i : y \succ_{S(v^i)} x\right\}\right| = 1$;
- matches against the sixth-placed team are discarded in the extra group: if $x \in X^{k+1}$, then $z \in X^i \setminus (X^i_x \cup \{x\})$ if and only if $y \succ_{S(v^i)} z$ for all $y \in X^i \setminus \{z\}$;
- the worst second-placed team is eliminated: $\mathcal{R}(V, x) = 0$ if $x \in X^{k+1}$ and $y \succ_{Q(V)} x$ for all $y \in X^{k+1} \setminus \{x\}$;
- the eight best second-placed teams are advanced to the next round: $\mathcal{R}(V, x) = 1$ if $x \in X^{k+1}$ and $\exists y \in X^{k+1} : x \succ_{Q(V)} y$.

Allocation rule $\mathcal{R}$ is fair due to the monotonicity of ranking methods S and Q together with $|X^i_v| = 5$ for all $x \in X^{k+1}$.

Proposition 5.1. *2018 FIFA World Cup qualification (UEFA) is not strategy-proof.*

Proof. The scenario presented in the proof of Theorem 4.1 shows that team Bulgaria $= x \in X^1$ can manipulate since there exist sets of group results $V = \{v^1, v^2, \dots, v^9\}$ and $\bar{V} = \{\bar{v}^1, v^2, \dots, v^9\}$ such that $\bar{v}^1 = v^1$ with the exception of $\bar{v}^1_1(y, x) = 1 > 0 = v^1_1(y, x)$, where team Luxembourg $= y \in X^1$ and $\mathcal{R}(V, x) = 0 < 1 = \mathcal{R}(\bar{V}, x)$.

Theorem 5.1 can also be applied because of Remark 5.1: the allocation rule is fair, the number of groups is $9 > 2$, $\mathcal{R}(V, x)$ can be 0 or 1 if team $x \in X^{k+1}$ is in the extra group, and, finally, $x \in X^i \cap X^{k+1}$ implies that $\left|\left\{y \in X^i : x \succ_{S(v^i)} y\right\}\right| = 4$, furthermore, $z \in X^i \setminus \{x\}$ if and only if $\left|\left\{y \in X^i : z \succ_{S(v^i)} y\right\}\right| = 1$. □

Theorem 5.1 can also be used to show that 2014 FIFA World Cup qualification (UEFA), which divided 53 teams into eight groups with six and one group with five, does not satisfy strategy-proofness. Note that Dagaev and Sonin (2013) have still verified the incentive incompatibility of this qualifier.

Now we state a positive result, a 'pair' of Theorem 5.1.

Theorem 5.2. *Let $(\mathcal{T}, \mathcal{R})$ be a qualification system such that $\mathcal{R}$ is a fair allocation rule and at least one of the following conditions hold:*

a) the number of groups is $k = 1$;

b) there is no need for an extra group;

c) there is no difference in the allocation of teams in the extra group, that is, $\mathcal{R}(V, x) = \mathcal{R}(V, y)$ for all $x, y \in X^{k+1}$;[13]

d) there exists no team $x \in X^i \cap X^{k+1}$ such that $\left| \left\{ y \in X^i : x \succ_{S(v^i)} y \right\} \right| \geq \ell + 1$ and $z \in X^i \setminus \{x\}$ if and only if $\ell = \left| \left\{ y \in X^i : z \succ_{S(v^i)} y \right\} \right| \geq 1$;

e) X^i_x is independent of v^i for all $x \in X^{k+1}$.

Then qualification system $(\mathcal{T}, \mathcal{R})$ satisfies strategy-proofness.

Proof. If there is only one group or there is no need for an extra group, monotonicity of S provides strategy-proofness.

If there is no difference in the allocation of teams in the extra group, then a team may improve its position in the extra group, but it has no incentives to cheat.

If all group matches are taken into account in the extra group or there exists no team ranked lower in the group than team x of the extra group such that matches against it are not discarded in the extra group, than team x cannot manipulate by changing the set of its matches to be ignored because of the monotonicity of S and Q.

If X^i_x is independent of v^i, then $\mathcal{R}(V, x) = \mathcal{R}(\bar{V}, x)$ under any sets of group results $V = \left\{ v^1, v^2, \dots, v^k \right\}$ and $\bar{V} = \left\{ v^1, v^2, \dots, \bar{v}^i, \dots, v^k \right\}$, so team x has no way to manipulate. □

6 On the strategy-proofness of some real-world qualification systems

At this point, it is known from Dagaev and Sonin (2013) and Proposition 5.1 that 2014 and 2018 FIFA World Cup qualifications (UEFA) are not strategy-proof. In the following, qualifications to the UEFA European Championships and FIFA World Cups in the European Zone will be analysed with respect to this property. We have devised Theorem 5.1 and Theorem 5.2 such that they will be enough to answer this question.

[13] Note that allocation rule $\mathcal{R}$ does not take seeding in play-offs into account. It is not a problem if play-offs are drawn randomly (like in the UEFA Euro 2000 qualifying) or based on an exogenous ranking of teams which is monotonic (like in the 2018 FIFA World Cup qualification (UEFA)). However, if, for example, the best half of all teams advanced to play-offs from the extra group are placed in Pot 1, then there is a difference in the allocation of teams in the extra group, despite it is not reflected by the allocation rule $\mathcal{R}$.

UEFA European Championship is hold every four years since 1960. The qualifications for the tournaments between 1960 and 1992 were organized without an extra group, so they was strategy-proof since condition b) of Theorem 5.2 is satisfied.

The 1996 qualifying consisted of seven groups with six and one group with five teams. The winner of each group along with the six best runners-up qualified directly, and the two worst runners-up met in a play-off. In the comparison of second-placed teams, only matches played against teams that finished first, third and fourth were considered. The second and third tie-breaking criteria were head-to-head number of points and goal difference, but they were not used recursively. Nevertheless, this qualification system was not strategy-proof because group and extra group ranking methods satisfied monotonicity, so Theorem 5.1 could be applied.

In 2000, the best runner-up qualified directly and the eight other were advanced to play-offs from the five groups of five teams and four groups of six teams. In the comparison of second-placed teams, matches played against teams that finished fifth or sixth were discarded. Consequently, it did not satisfy strategy-proofness as tie-breaking rules were the same as in the 1996 qualifying.

The 2004 and 2008 qualifying contained no extra group, they were strategy-proof because of condition b) of Theorem 5.2.

The 2012 qualification had six groups of six teams and three groups of five teams. The first-placed teams and the best second-placed team qualified. In the comparison of runners-up, matches against the sixth-placed team were not included. Tie-breaking was based on head-to-head results used recursively. Consequently, this qualification system did not satisfy strategy-proofness according to Theorem 5.1.

Theorem 5.1 also covers UEFA Euro 2016 qualifying with 53 teams drawn into eight groups of six and one group of five teams such that the group winners, runners-up, and the best third-placed team (such that the results against the sixth-placed team are discarded) directly qualify to the finals and the eight remaining third-placed teams contest play-offs to determine the last four qualifiers for the finals.

The first incentive incompatible FIFA World Cup qualifications for the European zone is the 1998 FIFA World Cup qualification (UEFA) with four groups of six teams and five groups of five teams. The runners-up would be ranked according to their records against the first, third and fourth-placed team in their groups, and the team with the best record qualified, while the others advanced to play-offs.

However, the 2002 FIFA World Cup qualification (UEFA) again satisfied strategy-proofness as all group winners qualified and among the runners-up, the one from Group 2 was drawn randomly to advance to an intercontinental play-off, while the other second-placed teams advanced to UEFA play-offs.

In the 2006 qualifying, group winners qualified to the World Cup, and the runners-up would be ranked such that results against the seventh placed team were ignored (there were three groups with seven and five with six teams) for the sake of fairness. The two best ranked second-placed teams qualified and the other six runners-up were drawn into play-offs. The second and third tie-breaking criteria were head-to-head number of points and goal difference, but they were not used recursively. Nevertheless, this qualification system does not satisfy strategy-proofness due to Theorem 5.1.

2010 FIFA World Cup qualification (UEFA) contained eight groups with six teams and one group with five. The nine group winners qualified directly, while the best eight second-placed teams contested play-offs. The second tie-breaking criterion was goal difference again. In determining the worst runner-up to be eliminated, the results against teams finishing last in the six team groups were not counted for consistency, opening a way to manipulation.

To summarize, on the basis of theoretical results presented in Section 5, at least nine recent incentive incompatible qualifications (to the 1996, 2000, 2012 and 2016 UEFA European Championships as well as to the 1998, 2006, 2010, 2014 and 2018 FIFA World Cups in the European Zone) can be identified.[14]

7 Discussion

Our findings described in the previous section carry a really frightening message for FIFA and UEFA: it has had a positive probability that a serious scandal occurs during a recent qualification to UEFA European Championships or FIFA World Cups, for example, in October 2017 as shown in a detailed example. In a sense, it would be even more disturbing than the 'Nichtsangriffspakt von Gijón' as one team would have an incentive not only to stop attacking, but to kick an own goal. Furthermore, it would be a more unfair case than Barbados vs. Grenada (1994 Caribbean Cup qualification) as the outcome of this match has not affected the qualification of any third team.

On the other hand, in the unlikely, yet possible scenario presented in the proof of Theorem 4.1, Luxembourg has practically no incentive to interfere in the manipulation of Bulgaria in order to prevent the elimination of Montenegro. Luxembourg may even interested in scoring a goal to be the fifth in the group. Fortunately, this situation has not materialized, and currently we do not know about any attempt to manipulate these qualifications in the way presented above.

Nevertheless, lack of dishonest behaviour is not an argument for diminishing the value of strategy-proofness, which can sometimes be satisfied without significantly changing the rules. For instance, in the 2018 FIFA World Cup qualification (UEFA), the root of the problem resides in the difference of group and second-places teams ranking by discarding the matches against the sixth-placed teams in the latter case. The greatest pity about this situation is that it could have been straightforward to avoid by UEFA ditching the strange policy of ignoring some group matches, since all groups would have six teams following the admission of Gibraltar and Kosovo. Yet they chose not to modify the rules. According to a recent UEFA News (UEFA, 2017), which was released on 10 October 2017, *after* the end of group stage: '*the exclusion of results against sixth-placed teams was retained to alleviate any possible imbalance between the qualifying groups caused by the late introductions of Gibraltar and Kosovo*'. While it is respectable that organizers have wanted to prevent some *mathematically unprovable* imbalances between the groups, it should be clear that they have sacrificed the much more clear and important issue of incentive compatibility.

However, it is necessary to show a mechanism providing strategy-proofness in order to argue against the rules of recent qualifications. While it was convenient in the case of 2018 FIFA World Cup qualification (UEFA), when all groups had the same number of

[14] We have also examined 2018 FIFA World Cup qualifiers of the other five confederations, AFC (Asian Football Confederation), CAF (Confederation of African Football), CONCAFAC (Confederation of North, Central American and Caribbean Association Football), CONMEBOL (South American Football Confederation) and OCF (Oceania Football Confederation). In the second round of AFC qualification, 40 teams were divided into eight groups of five teams such that the eight group winners and the four best runners-up advance to the third round. As a result of Indonesia being disqualified due to FIFA suspension, one group contained only four teams compared to five teams in all other groups. Therefore, the results against the fifth-placed team were not counted in the ranking of runners-up according to the related AFC regulation (AFC, 2015, Case 2).

teams, this condition cannot be guaranteed in all cases. So let us examine Theorem 5.2 and search for some strategy-proof qualification systems:

a) *There is only one group*: While it can implemented if the number of teams is not large (see 2018 FIFA World Cup qualification (CONMEBOL), the South American section of the 2018 FIFA World Cup qualification, which is also an excellent example for the cooperation of football governing bodies and the academic community (Durán et al., 2017)), it is certainly not an option with more than 10-12 teams.

b) *There is no need for an extra group*: It holds if all groups have the same number of teams, which may conflict with divisibility. However, we suggest to choose this solution when it can be implemented.

c) *There is no difference in the allocation of teams in the extra group*: It practically means that all second- or third-placed teams either qualify or advance to play-offs regardless that some groups may have more teams. For example, the allocation rule used in UEFA Euro 2004 qualifying (the top team in each group automatically qualifies, and the runners-up are paired for play-offs) is strategy-proof.

d) *Teams considered in the extra group have played no matches against teams ranked lower in the group, which are counted in the extra group* : If there are around 50 competing teams, it requires small group sizes (implying few group matches, therefore randomness is increased), standard group sizes with more rounds (which increases the number of matches to be played by a given team), or to discard many matches in the extra group, so it is not advised to use.

e) *Matches to be discarded in the extra group are independent of group results*: We think it could be the ultimate solution. Social choice theory usually wants to avoid the violation of anonymity at all costs, but see how groups are seeded: if n teams should be drawn into k groups, there would be k teams in Pot 1, k teams in Pot 2 and so on, until Pot m with $k < \ell < 2k$ (as in UEFA Euro 2008 qualifying) or with $\ell \leq k$ (as in 2018 FIFA World Cup qualification (UEFA)) is reached. Consequently, difference in group sizes is caused by Pot m, so it is fair and straightforward to discard matches against the team in Pot m for the ranking in the extra group, which immediately provides strategy-proofness according to Theorem 5.2.
In the case of 2018 FIFA World Cup qualification (UEFA) it means to fix in advance that matches played against teams in Pot 6 (Luxembourg, Andorra, San Marino, Georgia, Kazakhstan, Malta, Liechtenstein as well as the lately introduced Gibraltar and Kosovo) are discarded in the comparison of second-placed teams. Note that only Luxembourg and Georgia reached the fifth position in the qualification from Pot 6, so this policy makes not much difference in practice.
Nevertheless, a problem may arise when a team from Pot m should be considered in the extra group due its unexpectedly good performance.[15] While it is an unlikely scenario,[16] it can be immediately solved by discarding the match against the team in Pot $m - 1$ in the case of this team, which does not affect strategy-proofness.

[15] We are grateful to Dénes Pálvölgyi for spotting this issue.

[16] One of the greatest surprise occurred in 2016 UEFA Euro qualifying when Greece finished as the last team in its group despite it was drawn from Pot 1.

To summarize, we suggest to follow the subsequent mechanism in order to guarantee the strategy-proofness of a qualification system: (1) policy b) if the number of teams n can be divided by the number of groups k; (2) policy c) if n is not divisible by k but teams in the extra group can be treated uniformly (it is impossible if the extra group contains an odd number of teams); (3) policy e) if the first two policies cannot be implemented.

Another solution might be an artificial reduction of the number of teams for the sake of achieving equal group size. For instance, the weakest teams (e.g. Gibraltar, Liechtenstein, San Marino etc.) can be relegated to a special group, where they play against each other without the possibility of direct qualification. The winner of this extra group may advance to play-offs with runners-up (or third-placed teams), by playing with the best of them. Besides excluding manipulation, this solution has the further benefit of giving a chance for lower-ranked national teams, mainly composed of amateur players, to compete in their own league and enjoy more success than scoring some lucky goals against professional sportsmen.[17] It is also possible to organize a preliminary round for lower-ranked teams such as in the CEV qualification for the 2018 FIVB Volleyball Men's World Championship.

We hope this paper has reinforced our view that the scientific community and the sports industry should work more closely together in studying the effects of potential rules and rule changes even before they are implemented. For example, the governing bodies of major sports may invite academics to identify possible loopholes in proposed regulations in order to prevent serious scandals.

8 Conclusions

Design of appropriate sport ranking rules is an important theoretical problem of economics and operations research. Tournament organizers may face unpleasant situations when they miss analysing strategy-proofness. While manipulation is often a low-probability event, the potential costs can be enormous. We have demonstrated that decision makers have chosen a risky strategy in the case of some recent qualification tournaments to the UEFA European Championships and FIFA World Cups.

There are at least two possible directions for future research. First, a number of similar rules can be investigated from the perspective of incentive (in)compatibility. We plan to write some follow-up papers on this topic. Second, similarly to Berker (2014) and Lasek et al. (2016), the current theory-oriented investigation can be supplemented by estimating the probability of manipulation with the use of historical and Monte-Carlo simulated data.

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[17] This idea may partially inspired UEFA Nations League, which starts in September 2018 and is linked with UEFA Euro 2020 qualifying.

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Appendix

Table A.1: 2018 FIFA World Cup qualification – UEFA Group B

(a) Match results of the first eight matchdays
Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Switzerland	—	2-0	7 Oct	2-0	3-0	1-0
2	Portugal	10 Oct	—	3-0	5-1	6-0	4-1
3	Hungary	2-3	0-1	—	10 Oct	4-0	3-1
4	Faroe Islands	0-2	0-6	0-0	—	1-0	7 Oct
5	Andorra	1-2	7 Oct	1-0	0-0	—	0-1
6	Latvia	0-3	0-3	0-2	0-2	10 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
7 October 2017	Faroe Islands	Latvia	0-0
7 October 2017	Andorra	Portugal	0-3
7 October 2017	Switzerland	Hungary	2-0
10 October 2017	Hungary	Faroe Islands	2-0
10 October 2017	Latvia	Andorra	1-1
10 October 2017	Portugal	Switzerland	1-1

(c) Final standing with the runner-up results
Pos = Position; W = Won; D = Drown; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Switzerland	9	1	0	21	4	17	**28**
2	Portugal	8	1	1	32	5	27	**25**
3	Hungary	4	1	5	13	11	2	**13**
4	Faroe Islands	2	3	5	4	17	-13	**9**
5	Latvia	1	2	7	4	19	-15	**5**
6	Andorra	1	2	7	3	21	-18	**5**
2	Portugal	6	1	1	23	5	18	**19**

Table A.2: 2018 FIFA World Cup qualification – UEFA Group C

(a) Match results of the first eight matchdays
Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Germany	—	2-0	8 Oct	3-0	6-0	7-0
2	Northern Ireland	5 Oct	—	4-0	2-0	2-0	4-0
3	Azerbaijan	1-4	0-1	—	5 Oct	1-0	5-1
4	Czech Republic	1-2	0-0	0-0	—	2-1	8 Oct
5	Norway	0-3	8 Oct	2-0	1-1	—	4-1
6	San Marino	0-8	0-3	0-1	0-6	5 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
5 October 2017	Azerbaijan	Czech Republic	1-1
5 October 2017	Northern Ireland	Germany	0-1
5 October 2017	San Marino	Norway	0-2
8 October 2017	Czech Republic	San Marino	3-0
8 October 2017	Germany	Azerbaijan	2-0
8 October 2017	Norway	Northern Ireland	0-0

(c) Final standing with the runner-up results
Pos = Position; W = Won; D = Drown; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Germany	10	0	0	38	2	36	**30**
2	Northern Ireland	6	2	2	16	3	13	**20**
3	Czech Republic	3	4	3	14	10	4	**13**
4	Norway	3	2	5	10	16	-6	**11**
5	Azerbaijan	3	2	5	9	15	-6	**11**
6	San Marino	0	0	10	2	43	-41	**0**
2	Northern Ireland	4	2	2	9	3	6	**14**

Table A.3: 2018 FIFA World Cup qualification – UEFA Group D

(a) Match results of the first eight matchdays
Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Serbia	—	1-1	2-2	3-2	9 Oct	3-0
2	Wales	1-1	—	9 Oct	1-0	1-1	4-0
3	Republic of Ireland	0-1	0-0	—	1-1	1-0	6 Oct
4	Austria	6 Oct	2-2	0-1	—	1-1	2-0
5	Georgia	1-3	6 Oct	1-1	1-2	—	1-1
6	Moldova	0-3	0-2	1-3	9 Oct	2-2	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
6 October 2017	Georgia	Wales	0-1
6 October 2017	Austria	Serbia	0-0
6 October 2017	Republic of Ireland	Moldova	2-0
9 October 2017	Moldova	Austria	1-2
9 October 2017	Serbia	Georgia	1-1
9 October 2017	Wales	Republic of Ireland	1-0

(c) Final standing with the runner-up results
Pos = Position; W = Won; D = Drown; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Serbia	5	5	0	18	8	10	**20**
2	Wales	5	5	0	14	5	9	**20**
3	Republic of Ireland	4	4	2	11	7	4	**16**
4	Austria	3	4	3	12	11	1	**13**
5	Georgia	0	6	4	9	14	-5	**6**
6	Moldova	0	2	8	5	24	-19	**2**
2	Wales	3	5	0	8	5	3	**14**

Table A.4: 2018 FIFA World Cup qualification – UEFA Group E

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Poland	—	8 Oct	3-2	3-1	2-1	3-0
2	Montenegro	1-2	—	5 Oct	1-0	4-1	5-0
3	Denmark	4-0	0-1	—	8 Oct	1-0	4-1
4	Romania	0-3	1-1	0-0	—	1-0	5 Oct
5	Armenia	5 Oct	3-2	1-4	0-5	—	2-0
6	Kazakhstan	2-2	0-3	1-3	0-0	8 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
5 October 2017	Armenia	Poland	1-5
5 October 2017	Montenegro	Denmark	0-0
5 October 2017	Romania	Kazakhstan	2-0
8 October 2017	Denmark	Romania	1-1
8 October 2017	Kazakhstan	Armenia	1-0
8 October 2017	Poland	Montenegro	1-1

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drown; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).
Note: Montenegro is ranked above Denmark due to FIFA (2016, Article 20.6d) because it has obtained 4 points against Denmark, while Denmark has obtained only 1 point against Montenegro.

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Poland	7	2	1	24	13	11	**23**
2	Montenegro	5	3	2	19	8	11	**18**
3	Denmark	5	3	2	19	8	11	**18**
4	Romania	3	4	3	11	9	2	**13**
5	Armenia	2	0	8	9	25	-16	**6**
6	Kazakhstan	1	2	7	5	24	-19	**5**
2	Montenegro	3	3	2	12	6	6	**12**

Table A.5: 2018 FIFA World Cup qualification – UEFA Group F

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	England	—	2-1	5 Oct	3-0	2-0	2-0
2	Slovakia	0-1	—	1-0	3-0	4-0	8 Oct
3	Slovenia	0-0	1-0	—	8 Oct	4-0	2-0
4	Scotland	2-2	5 Oct	1-0	—	1-1	2-0
5	Lithuania	8 Oct	1-2	2-2	0-3	—	2-0
6	Malta	0-4	1-3	0-1	1-5	5 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
5 October 2017	England	Slovenia	2-1
5 October 2017	Malta	Lithuania	0-1
5 October 2017	Scotland	Slovakia	0-0
8 October 2017	Lithuania	England	1-3
8 October 2017	Slovakia	Malta	3-0
8 October 2017	Slovenia	Scotland	1-0

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drown; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	England	8	2	0	21	5	16	**26**
2	Slovakia	6	1	3	17	6	11	**19**
3	Slovenia	5	2	3	12	6	6	**17**
4	Scotland	4	3	3	14	11	3	**15**
5	Lithuania	2	2	6	8	21	-13	**8**
6	Malta	0	0	10	2	25	-23	**0**
2	Slovakia	4	1	3	11	5	6	**13**

Table A.6: 2018 FIFA World Cup qualification – UEFA Group G

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Spain	—	3-0	6 Oct	4-1	4-0	8-0
2	Italy	1-1	—	2-0	1-0	6 Oct	5-0
3	Albania	0-2	9 Oct	—	0-3	2-1	2-0
4	Israel	9 Oct	1-3	0-3	—	0-1	2-1
5	FYR Macedonia	1-2	2-3	1-1	1-2	—	9 Oct
6	Liechtenstein	0-8	0-4	0-2	6 Oct	0-3	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
6 October 2017	Italy	FYR Macedonia	2-0
6 October 2017	Liechtenstein	Israel	0-1
6 October 2017	Spain	Albania	3-1
9 October 2017	Albania	Italy	1-2
9 October 2017	Israel	Spain	0-3
9 October 2017	FYR Macedonia	Liechtenstein	2-1

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drown; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Spain	9	1	0	36	3	33	**28**
2	Italy	8	1	1	23	8	15	**25**
3	Albania	4	1	5	12	14	-2	**13**
4	Israel	4	0	6	10	17	-7	**12**
5	FYR Macedonia	3	1	6	12	17	-5	**10**
6	Liechtenstein	0	0	10	2	36	-34	**0**
2	Italy	6	1	1	14	8	6	**19**

Table A.7: 2018 FIFA World Cup qualification – UEFA Group H

(a) Match results of the first eight matchdays
Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017
Bosnia and Herz. stands for Bosnia and Herzegovina

Position	Team	1	2	3	4	5	6
1	Belgium	—	4-0	1-1	10 Oct	8-1	9-0
2	Bosnia and Herz.	7 Oct	—	0-0	2-0	5-0	5-0
3	Greece	1-2	1-1	—	2-0	0-0	10 Oct
4	Cyprus	0-3	3-2	7 Oct	—	0-0	3-1
5	Estonia	0-2	10 Oct	0-2	1-0	—	4-0
6	Gibraltar	0-6	0-4	1-4	1-2	7 Oct	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
7 October 2017	Gibraltar	Estonia	0-1
7 October 2017	Bosnia and Herzegovina	Belgium	0-3
7 October 2017	Cyprus	Greece	0-1
10 October 2017	Belgium	Cyprus	3-1
10 October 2017	Estonia	Bosnia and Herzegovina	1-2
10 October 2017	Greece	Gibraltar	3-0

(c) Match results of the first eight matchdays
Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Belgium	9	1	0	40	4	36	**28**
2	Greece	5	4	1	15	5	10	**19**
3	Bosnia and Herzegovina	5	2	3	21	11	10	**17**
4	Cyprus	3	1	5	9	16	-7	**11**
5	Estonia	3	2	5	8	19	-11	**11**
6	Gibraltar	0	0	10	3	41	-38	**0**
2	Greece	3	4	1	8	4	4	**13**

Table A.8: 2018 FIFA World Cup qualification – UEFA Group I

(a) Match results of the first eight matchdays

Position is given according to the eight matches already played
Home team is in the row, away team (represented by its position) is in the column
Dates are given for the matches to be played on the last two matchdays in 2017

Position	Team	1	2	3	4	5	6
1	Croatia	—	2-0	1-1	1-0	6 Oct	1-0
2	Iceland	1-0	—	2-0	2-0	3-2	9 Oct
3	Turkey	1-0	6 Oct	—	2-2	2-0	2-0
4	Ukraine	9 Oct	1-1	2-0	—	1-0	3-0
5	Finland	0-1	1-0	9 Oct	1-2	—	1-1
6	Kosovo	0-6	1-2	1-4	6 Oct	0-1	—

(b) Hypothetical but reasonable match results of the last two matchdays

Date	Home team	Away team	Result
6 October 2017	Croatia	Finland	2-0
6 October 2017	Kosovo	Ukraine	0-2
6 October 2017	Turkey	Iceland	2-1
9 October 2017	Finland	Turkey	0-1
9 October 2017	Iceland	Kosovo	1-0
9 October 2017	Ukraine	Croatia	0-0

(c) Final standing with the runner-up results

Pos = Position; W = Won; D = Drown; L = Loss; GF = Goals for; GA = Goals against; GD = Goal difference; Pts = Points. All teams have played 10 matches.
Last row contains the second-placed team's results, adjusted for the ranking of the runners-up (matches played against the 6th team are discarded) according to FIFA (2017).

Pos	Team	W	D	L	GF	GA	GD	**Pts**
1	Croatia	6	2	2	14	3	11	**20**
2	Turkey	6	2	2	14	9	5	**20**
3	Ukraine	5	3	2	13	7	6	**19**
4	Iceland	6	1	3	13	9	4	**19**
5	Finland	2	1	7	6	12	-6	**7**
6	Kosovo	0	1	9	3	23	-20	**1**
2	Turkey	4	2	2	8	8	0	**14**